

Problem Set I: Due Tuesday, January 17, 2012

This is a review problem set on 218A material, and basic statistical mechanics. It consists of closed book exam problems from a similar course given at KAIST, in Korea, this fall. Try to do it closed book, in three hours. Then, do the rest open book.

We will discuss it next week, in the problem session.

1.)

- (a.) Derive the equation for near-equilibrium relaxation of a stable plasma with thermal fluctuations. Discuss the physics of each term. You may find it useful to note that the transformed discreteness correlation function is:

$$\langle \tilde{f}\tilde{f} \rangle_{k,\omega} = \frac{2\pi}{n} \delta(\omega - kv) \delta(v_1 - v_2) \langle f \rangle$$

Assume 1D.

- (b.) Consider a 1D plasma with Maxwellian electrons (Te) and ions (Ti). Also assume the electrons carry a finite net momentum $n_0 m_e u_0$, but the waves in the system are damped. Calculate, using your result for (a.), the rate at which electron momentum decays.
- 2.) Calculate Q , the rate of collisionless power dissipation, for the electrostatic electron plasma (Langmuir) wave with $\omega = \omega(k)$. You may assume 1D, and a Maxwellian $\langle f \rangle$. Calculate $\omega(k)$ explicitly.

Note: Recall $Q = \omega(k) \epsilon_{IM} \frac{|E_k|^2}{8\pi}$, where $\epsilon_{IM} = \epsilon_{IM}(k, \omega(k))$.

- 3.) Calculate the intensity of the (random) thermal fluctuation force \tilde{f} which acts on a particle moving in a fluid at temperature T . Assume the particle's motion is described by $\ddot{x} + \gamma\dot{x} = \tilde{f}/m$, where \tilde{f} is delta correlated.

4.)

| | Landau Collision Integral | Balescu-Lenard |
|----------------------------------|---------------------------|----------------|
| Scenario – Physical Idea | | |
| Assumption on Correlation | | |
| Collective Effects and Screening | | |
| Nature of Interactions | | |

N.B. Structure your answers as a comparison – contrast of the two theoretical formulations.

- 5.)
- (a.) Consider a hard sphere gas with $\langle f \rangle$ a local Maxwellian where $T = T(x)$. Treat the potential of interaction as having range a and strength u_0 . What conditions (scale in equality) must be satisfied in order that a Chapman – Enskog approach to calculation of transport coefficients is justified? Explain your answers.
- (b.) Estimate the inter-particle collision frequency.
- (c.) Using a Krook model and your result from (b.), calculate the collisional heat flux and rate of entropy production.
- 6.) Calculate the deflection in momentum of a test particle of velocity \underline{v} from a field particle with velocity \underline{v}' in a plasma with dielectric function $\epsilon(k, \omega)$. Here:
- i.) assume the deflection is small and the test particle moves along a linear trajectory.
- ii.) assume the field particle generates a screened potential.
- 7.) Consider a current-driven system in 1D. Electrons are Maxwellian, with centroid at u_0 , temperature T_e . Ions have temperature T_i . Assume the system is collisionless.
- (a.) Derive an expression for the mean electric field required to maintain the electron mean velocity u_0 . Your answer should depend on \tilde{E} and \tilde{f} .
- (b.) Calculate the general condition for stability of this system. Do not assume $k^2 \lambda_D^2 \ll 1$. Be as explicit as possible. (You can ignore the external field here.)
- (c.) Assuming a spectrum of unstable CDIA waves, derive an expression for the mean electric field required to maintain a stationary state. You should leave your answer as a function of u_0 and the wave spectrum.
- (d.) Prove that total resonant particle energy and total wave energy are conserved here, at the level of quasi-linear theory. N.B. You must consider both resonant electrons and resonant ions.

- 8.) Consider a 1D system of a cold beam, with density n_0 and velocity V_0 interacting with a cold plasma of density n_0 .
- (a.) Calculate the wave energy of modes of the beam only. Explain your results.
 - (b.) Calculate the condition for instability, assuming $n_b = n_0$. Explain the physics of your result. Hint: Choose your frame carefully!
 - (c.) Now take $n_b \ll n_0$. Estimate when instability occurs. Explain the physics of your result.
- 9.)
- (a.) Derive the Fokker-Planck/Schmoluchowski equation for the evolution of $n(z)$ - the density of sediment - in a fluid at temperature T in a gravitational field. Show all steps in the derivation.
 - (b.) What is the stationary profile? Explain the physics of your result.